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INCOMPLETE CONTRACTS AND RENEGOTIATION

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by

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1. INTRODUCTION

One of the most serious problems facing the writers of a contract is that of specifying all relevant aspects of contractual performance. In particular, it may be prohibitively costly to indicate the precise actions that each party to the contract should take in every conceivable eventuality. Because of these "transaction costs", the parties are in practice likely to end up writing a highly incomplete contract.

A formal distinction can be drawn between problems arising from contractual incompleteness and those arising from asymmetries of information, although the overlap between the two is considerable. In the latter case, certain contingent statements are infeasible because the state of the world is not observed by all parties to the contract. In the case of contractual incompleteness, on the other hand, the parties may have the same information; what prevents the use of a complete contingent contract is the cost of processing and using this information in such a way that the appropriate contingent statements can be included and implemented. These "transaction costs" (some of which go under the heading of bounded rationality) may also limit the complexity of contracts.

Problems of incomplete contracting have received relatively little attention from economists, presumably because of the difficulty of formalizing the notion of transaction costs. These problems have, however, been recognized as having important implications for the organization of economic activity (see, e.g., Williamson (1979)) and lie at the center of the

legal literature on contracts and of many contractual cases which come before the courts (see, e.g., Dawson, Harvey and Henderson (1982)).

While the economics literature on incomplete contracts is small, some attempts have been made to analyze them. Among others, the papers by Shavell (1979) (1984), Rogerson (1984), Dye (1981), Grout (1984), Hall and Lazear (1984), Tirole (1985), and Weitzman (1981) should be mentioned. Because of the difficulties of formalizing the notion of transaction costs, however, these papers have tended simply to assume that certain contingent statements have a fixed cost associated with them -- in the extreme case this cost is infinite and so the statements cannot be written into the contract at all. In Grossman and Hart (1984), a limited attempt was made to explain why some contingent statements are infeasible. In that paper, a model is considered of trade between a buyer and seller of an item. (The model is similar in spirit to one in Hall and Lazear (1984).) From an efficiency point of view, the level of trade, q, and also possibly the payment p of the buyer to the seller, should depend on the buyer's benefit, v, of having the item and the seller's cost c of supplying it. Suppose, however, v and c are not observable to the enforcers of the contract, i.e., the courts. Moreover, although the values of v and c are determined by the state of the world, ω, which <u>is</u> publically observable, suppose ω is too complicated to be described unambiguously in the contract.[1] Then contingent statements of the form "$q=\hat{q}$ if $\omega=\hat{\omega}$ or $(v,c)=(\hat{v},\hat{c})$" are infeasible and so the contract is necessarily incomplete.

In spite of this incompleteness, it may be possible for the parties to make both q and p sensitive to (v,c) through revision and/or renegotiation

of the contract once ω is realized. This possibility, which was not considered in Grossman and Hart (1984), is the main concern of the present paper. That is, we shall analyze the form of an optimal incomplete contract under the assumption that the parties to the contract can revise and/or renegotiate the contract as new information becomes available.[2]

Given rational expectations by the parties, the fact that revisions will occur will affect the form of the initial contract. Less obvious, perhaps, is the fact that it will be in the interest of the two parties to try to constrain or limit in the original contract the sorts of revisions that can later take place. That is to say, the parties face the problem of designing an optimal revision game to be played once the state of the world, ω, is realized in order to yield final quantities and prices which are appropriately sensitive to ω. This game or mechanism design problem will be the focus of much of the paper. It should be noted that our approach to modelling renegotiation is rather different from that found in the literature on noncooperative approaches to bargaining (see, for example, Rubinstein (1981)).

In carrying out our analysis, we ignore other transaction costs reasons for contractual incompleteness, such as bounded rationality. This neglect may be significant since bounded rationality may limit the types and complexity of revision games that the buyer and seller can conceive of. We ignore the complexity issue, not because we think it is unimportant, but because we do not know how to deal with it at a formal level (for a discussion of complexity, see, e.g., Simon (1981)). We also believe that in at least some situations, the parties to a contract may be sufficiently

sophisticated to consider the type of revision processes we study, i.e., it is the inability to describe ω which really constitutes the major "transaction cost".

The paper is organized as follows. The model is set out in the next Section, and the critical assumptions concerning the timing and transmission of messages are discussed. In Section 3, the class of possible trading prices is found when messages cannot be verified by outsiders. Section 4 does the same, but under the assumption that messages can be verified. These results are used for two different applications in Section 5. Conclusions are in Section 6.

2. THE MODEL

We consider the (long-term) relationship between a buyer and a seller of an item or good. The buyer and seller write a contract at some initial date 0 which specifies the conditions of trade between them in the future. To simplify, we assume that all trade occurs at a single date, date 2. At date 2, either one unit is traded, or zero. The buyer's valuation of one unit at date 2 is given by the random variable v and the seller's cost by the random variable c. These random variables are determined by the state of the world, ω, i.e. we have

$$(v,c) = f(\omega)$$

where $\omega \varepsilon \Omega$, the set of all states of the world, and f is some function mapping $\Omega \to R^2$. To simplify, we suppose that the set Ω is finite. The state ω is assumed to be determined at date 1, and is publically observable. The buyer

and seller are supposed to know the function f and hence each can deduce the pair (v,c) at date 1, i.e. there is symmetric information between them at date 1. The period between date 1 and date 2 can be used by the parties to revise and/or renegotiate the initial contract.[3] The sequence of events is illustrated below:

Date 0 — Date 1 — Date 2 — |

Contract Signed

(v,c) learned by buyer and seller

Trade?

Disputes?

Revision ←and/or renegotiation→ of the contract

Our assumption is that if the good is traded after date 2, its value drops to zero (and it is not ready to be traded before date 2). Any disputes by the buyer and seller over whether the contract has been carried out occur after date 2 and are resolved by a court. In fact, as we shall see, in equilibrium, no disputes actually occur. Finally, the distribution of v and c is assumed to be common knowledge at date 0.

We suppose that the buyer and seller must make some specific investments after date 0 in order to ensure that later trade is mutually beneficial (although we shall not always be explicit about these). As a result of these specific investments, the buyer and seller are to some extent locked into each other after date 0. In fact, to simplify matters, we suppose that the lock-in is complete, in the sense that by the time date 1 arrives, neither the buyer nor the seller can trade with any other party. At date 0, in contrast, we suppose that there is no lock-in, in the sense that there are

many similar sellers (resp. buyers) with whom the buyer (resp. the seller) can form a relationship, and that the division of the ex-ante surplus between this buyer and seller is determined in a competitive market for contracts.

With this background, let us now turn to the form of an optimal contract at date 0.[4] Let $q = 0$ or 1 be the amount traded between the buyer and seller at date 2. Clearly for trade to be efficient, we must have

(2.1) $$q = 1 \iff v \geq c.$$[5]

The first point to note is that this trading rule can be implemented by the parties <u>without</u> writing a contract at date 0; they can simply wait until date 2 and bargain (efficiently) then about how the date 2 gains from trade should be divided (note, however, that in order to realize the right ex-ante division of surplus, a sidepayment will have to be made from one party to the other at date 0). The problem with this solution is that the ex-post distribution of the surplus which it generates may be undesirable from an ex-ante point of view. This will, in particular, be the case if the parties are <u>risk-averse</u> or if they take <u>actions</u> which affect the probability distribution of v and c (in which case the ex-post distribution of surplus will influence these actions). Throughout the paper, we will have one of these cases in mind, which is why the "no date 0 contract" solution will be suboptimal.

The "first-best" contract would specify the trading rule (2.1) and a price $p(v,c)$ that the buyer must pay the seller in state ω, where $f(\omega) = (v,c)$, and $p(\cdot)$ is chosen to achieve an optimal division of the ex-post surplus. In order to enforce such a contract, the courts would have to be able to rule on whether a particular state $\omega = \hat{\omega}$ had occurred, which requires that the parties describe ω in detail in the date 0 contract. Our

assumption is that it is prohibitively costly to provide the relevant detail about $\hat{\omega}$, and so the first-best contract is infeasible. That is, imagine that ω consists of a list of factors, such as conditions in related input and output markets, the level of GNP, certain exchange rates, whether a strike is likely in this industry, whether a war appears imminent in the Middle-East, etc., etc. Some of these factors can be thought about and described in advance, but others surely cannot. Also, even if all these factors can be anticipated and described, the parties may not know the function f at date 0, i.e. they may not know how all these factors map into their own value and cost -- it may only be after some time of working closely together that this mapping is learned by them. In this case although contingent statements about ω at date 0 are possible, they are not useful since the relevant variables v,c with respect to which the statements should be made are only distantly related to ω.

So either because ω cannot be described or because f is not known, contingent statements about v,c are assumed to be impossible. To summarize, the parties realize at date 0 that they will both learn (v,c) before they trade, but they also recognize that outsiders, such as the courts, will not observe (v,c) and hence cannot enforce trading rules or prices which depend directly on (v,c).[6] As we shall see, the parties may be able to overcome this ignorance of outsiders by sending messages which reveal part of (v,c) to the courts.[7]

We have argued that the courts cannot observe (v,c). What can they observe? We suppose that in the event of a dispute, all they can determine is (a) whether trade occurred or not, i.e. whether q = 0 or 1; (b) how much

the buyer paid the seller; (c) certain messages, letters or written documents that were exchanged between the buyer and seller between date 1 and date 2. If we ignore (c) for a moment, the implication of (a) and (b) is that a contract between the buyer and seller can specify only two prices, p_0 and p_1, where p_i is the price the buyer must pay the seller if $q = i$ $(i=0,1)$. The effect of (c) is to allow p_i to depend on some messages $\underset{\sim}{m}$ exchanged by the buyer and seller between dates 1 and 2. Hence the contract can specify price <u>functions</u> $p_0(\underset{\sim}{m})$, $p_1(\underset{\sim}{m})$ rather than just numbers p_0, p_1.[8]

Implicit in (a)-(c) is the assumption that the courts cannot determine <u>why</u>, if $q=0$, trade does not occur; that is, the courts cannot distinguish between non-trade due to the seller being unwilling to supply and due to the buyer being unwilling to take delivery.

Given the initial contract, and once messages have been exchanged, trade will only occur at date 2 if <u>both</u> parties are willing. The buyer incurs no penalties (over and above having to pay $p_0(\underset{\sim}{m})$ to the seller) from not accepting delivery, and neither does the seller from not supplying. That is, trade will occur if and only if

$$v - p_1(\underset{\sim}{m}) \geq -p_0(\underset{\sim}{m}),$$

(2.2) and

$$p_1(\underset{\sim}{m}) - c \geq p_0(\underset{\sim}{m}).$$

The first part of (2.2) says that the buyer is better off with $q=1$ than $q=0$, and the second part says that the same is true of the seller. (2.2) can be written more compactly as

(2.3) $\quad q=1 \iff v \geq p_1(\underset{\sim}{m}) - p_0(\underset{\sim}{m}) \geq c.$

This same trading rule, but without the messages, is considered in Hall and Lazear (1984) and Grossman and Hart (1984).[9/]

Before proceeding, we must mention an important implicit assumption that we have made. This is that it is impossible for the buyer and seller to include a <u>third party</u> in the date 0 contract, with this third party acting as a financial wedge between the buyer and seller. In this case the price paid by the buyer in a particular state does not have to equal the price received by the seller, with the third party making up the difference. In Appendix A, we shall give some reasons why three party contracts of this sort may be difficult to implement.

Returning to the two party situation, let us consider now the exchange of messages between dates 1 and 2. To fix ideas, imagine that the date 0 contract specifies only two prices p_0, p_1, rather than two price functions. If $v \geq p_1 - p_0 \geq c$ at date 2, trade will take place at these prices. But, suppose that $v>c$ but either $p_1-p_0>v$ or $c>p_1-p_0$. Then even though there are gains from trade, they will not be realized under the contract. Hence the exchange of messages can be seen as a way of <u>revising</u> the contract.

Put this way, the exchange of messages (or proposals) is clearly very similar to what occurs in the classical bargaining problem where two parties are deciding how to divide a pie. There is now a sizeable literature which analyzes this bargaining problem as a noncooperative extensive form game

(see, e.g. Rubinstein (1981)). Unfortunately, the particular models which have been analyzed in this literature have features which seem unsatisfactory for the purpose at hand. In particular, (a) they do not directly address the issue of what happens when there is already a contract in place; (b) they tend to assume that the agents alternate in the making of proposals, which seems rather arbitrary; (c) they do not consider the issue of how it is verified that an agreement has been reached or how the agreement is enforced. Thus although influenced by these analyses, we have developed our own model of the way revisions occur.

We imagine that the time between dates 1 and 2 is divided up into a number of "days", as in Figure 1. Messages or "letters" are sent by a totally reliable "mail" service and take a day to arrive. There is one collection and one delivery of mail a day (for both the buyer and seller). Delivery of day (i-1)'s mail occurs before collection of day i's mail. The buyer and seller can send each other messages on the same day (i.e. simultaneously), and can send several messages at the same time.

Day 1 Day 2 Day 3 Day 4 ... Day n

Date 1 Date 2

Figure 1

In Figure 1, messages can be sent on days 1-n. It is supposed that messages sent on the last day, day n, arrive before the seller and buyer decide whether to trade at date 2.

We assume throughout:

(*) Messages cannot be forged. That is, the buyer, say, cannot claim that the seller sent him a message when in fact he really didn't. It can be imagined that all messages are signed by the person who sent them and signatures cannot be forged.

Although messages cannot be forged, it does not follow from this that the recipient of a message cannot deny that he received it. In fact, we shall distinguish between two cases:

(A) There is no (public) record that any particular message was sent by one party to another. If one party receives a message, he can choose to reveal it in the event of a dispute, but is under no obligation to do so since he can always deny that he received it.

(B) A public record exists of all messages sent (and received). Hence a party cannot deny receipt of a message.

Case (A) corresponds to the usual mail service. Case (B) corresponds perhaps to the case of telegrams, where the telegraph company (assumed to be totally reliable and honest) keeps a record of the contents of each telegram received or gives the sender a copy.[10]

It turns out that the form of the optimal contract is very sensitive to whether (A) or (B) applies. In the next section, therefore, we analyze Case

(A) and in the following section Case (B). In both sections we make one further assumption:

(**) There is nothing to stop the two parties agreeing at any time to tear up, or rescind, the date 0 contract and write a new one.

(**) seems very reasonable. First, it corresponds to the way contracts are treated under the law. Secondly, it is hard to see how the parties could constrain themselves in advance not to revise a contract (but see footnote 14).

3. CASE (A): SENDING A MESSAGE CANNOT BE VERIFIED

The task facing the buyer and seller at date 0 is to design a message or revision game to be played at date 1, which will yield trade and price outcomes that are appropriately sensitive to the realized pair (v,c). In principle, this game can be very complex: it may involve many moves by each player, some of which are simultaneous, some of which are sequential. In fact, in Case (A), the game really consists of two sub-games, one of which is the pure message game played between date 1 and date 2, and the other the dispute game played after date 2. In this dispute game, each party decides which of the messages received from the other party to reveal to the court (a decision which may depend on the strategy which the party expects the other party to follow in this regard). This dispute game is played <u>after</u> q has been chosen, its concern being the price that should be paid -- p_1 if q was equal to 1 and p_0 if q was equal to 0. Note that both sub-games are games of complete information, since v,c are known to both parties at date 1.

A natural approach is to study the (perfect) Nash equilibria of each sub-game and, in light of this, to consider what is the optimal message game for the buyer and seller to select at date 0. At first sight this exercise seems daunting, given the potentially very large number of games which might be played. It turns out, however, that, in Case (A), by virtue of (**), the possibilities at the disposal of the two parties are actually extremely limited.

First, note that in Case (A), neither party can be forced to send a message; that is, the date 0 contract cannot, for example, penalize the buyer for not sending a message by raising p_0 and p_1, since the seller can (and will) then increase his profit by denying that the message was received. This is one way in which a third party could be helpful: the buyer's penalty for not sending a message could be paid to the third party rather than to the seller, which would remove the seller's incentive to deny receipt. In section 2, however, we ruled out third parties, an assumption which is justified in Appendix A.

Given that the parties cannot be forced to send messages, the contract must specify the prices p_0, p_1 which will apply if no messages are sent. One can think of p_0, p_1 as being "default" or "status quo" contract prices, i.e. prices that rule in the absence of revisions. We now show that, in Case (A), once the buyer and seller have chosen p_0 and p_1, the whole of the rest of the contract is determined. The precise way in which this occurs is described in Proposition 1.

<u>Proposition 1</u> Let $(\hat{p}_0, \hat{p}_1)$ be the prices which the date 0 contract specifies

will apply if no messages are sent between dates 1 and 2. Then, in Case (A), the trading rule and prices which will obtain at date 2 are as follows:

(1) If $v<c$, $q=0$ and the buyer pays the seller $\hat{p}_0$.

(2) If $v \geq \hat{p}_1-\hat{p}_0 \geq c$, $q=1$ and the buyer pays the seller $\hat{p}_1$.

(3) If $v \geq c > \hat{p}_1-\hat{p}_0$, $q=1$ and the buyer pays the seller $\hat{p}_0+c$.

(4) If $\hat{p}_1-\hat{p}_0 > v \geq c$, $q=1$ and the buyer pays the seller $\hat{p}_0+v$.

Proposition 1 is illustrated in Figure 2, where $k \equiv \hat{p}_1-\hat{p}_0$, and v varies between $\underline{v}$ and $\bar{v}$, and c between $\underline{c}$ and $\bar{c}$.

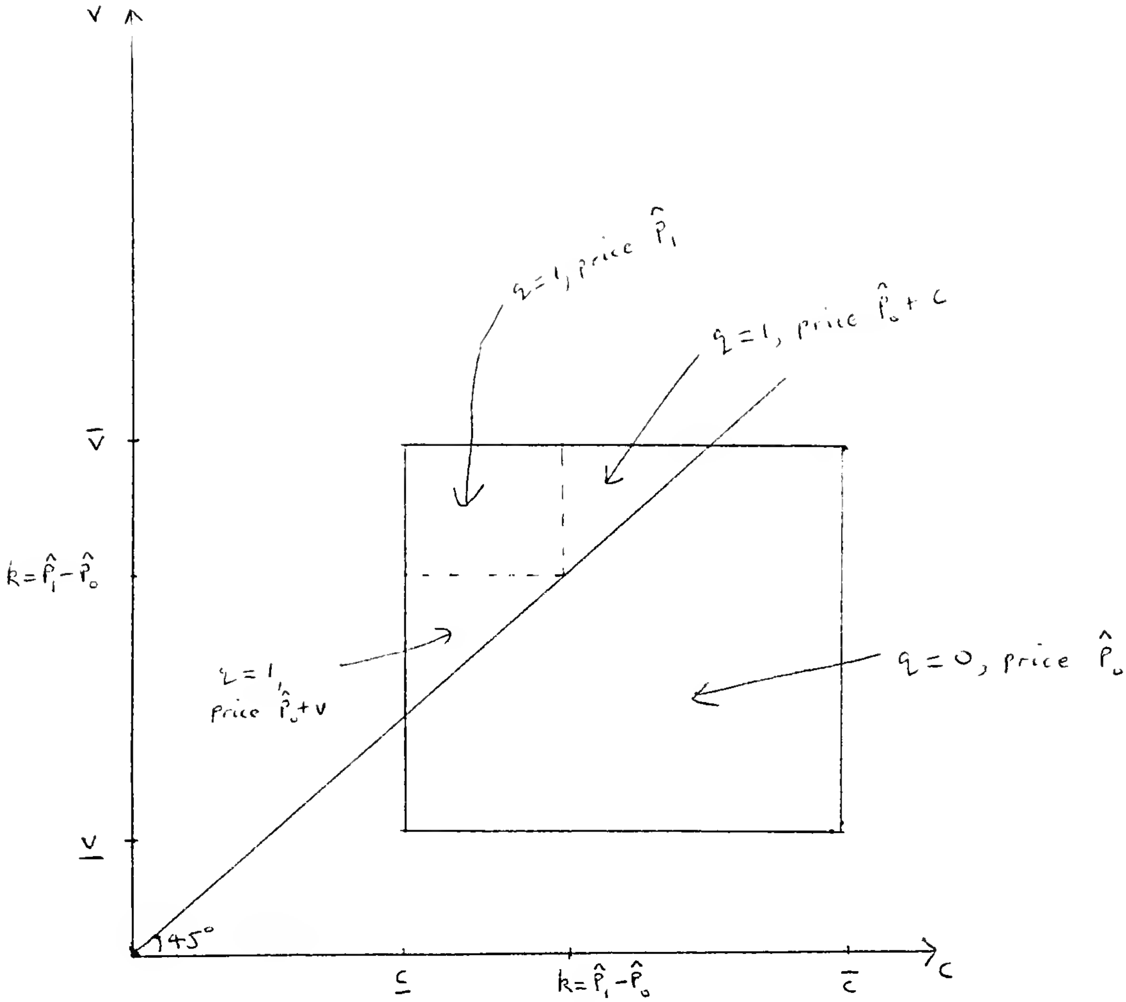

Figure 2

Note that trade occurs if and only if it is efficient ex-post, i.e. if and only if $v \geq c$. In the no-trade region, the price is constant at $\hat{p}_0$. The trade region is divided up into three parts. In the dotted box, the initial contract price $\hat{p}_1$ rules. In the triangle to the North-East of this, the price $\hat{p}_0+c$ rules. And in the triangle to the South-West of this, the price $\hat{p}_0+v$ rules.

We remarked before that our assumption is that the division of ex-ante surplus betweeen the buyer and seller is determined in the date 0 market for contracts. Let $\bar{U}$ be the market equilibrium expected utility level for the seller. Then given k, $\hat{p}_0$ (and hence $\hat{p}_1=\hat{p}_0+k$) will be determined so as to satisfy the seller's expected utility constraint. In other words, Proposition 1 tells us that, given a fixed division of the ex-ante surplus, the buyer and seller have not two degrees of freedom $\hat{p}_0, \hat{p}_1$, but in fact only <u>one</u>, k, in structuring their contract.

We now give a (somewhat informal) proof of Proposition 1. Consider first the region $v<c$. Whatever messages are sent and revealed, the parties know that (2.3) cannot be satisfied. Hence they know that trade will not occur at date 2, and the relevant price is $p_0(\underset{\sim}{m})$. This means that the buyer and seller are playing a zero-sum game over the price p_0. Given that each can hold the other to the price $\hat{p}_0$ by sending no messages, and revealing none from the other party in the event of a dispute, it follows that the unique Nash equilibrium price is $\hat{p}_0$. That is, over the region $v<c$, $p_0 \equiv \hat{p}_0$ is independent of (v,c).

A similar argument applies to the dotted box region,

(3.1) $$v \geq \hat{p}_1 - \hat{p}_0 \geq c.$$

Suppose first the buyer and seller send no messages. Then when date 2 arrives, in view of (3.1), the seller will wish to supply and the buyer will wish to accept the good, and so trade will occur at the price $\hat{p}_1$. That is, it is <u>feasible</u> for trade to occur without any revisions being made to the

contract. Might trade take place at any other price? The answer is no, since, if the price were higher, as a result of certain messages or revisions, the buyer could do better by sending no messages and revealing none from the seller, which would guarantee him a price of $\hat{p}_1$. Similarly, if the price were lower than $\hat{p}_1$, the seller could do better by sending no messages and revealing none from the buyer. That is, each party can hold the other to trade at the price $\hat{p}_1$, and so this must be the unique perfect Nash equilibrium.

Although this argument is very similar to that given in the no-trade region $v<c$, it is in fact a bit more subtle. The game is not zero sum anymore since even though trade can take place at the unrevised contract prices $\hat{p}_0, \hat{p}_1$, the parties may exchange messages in such a way that the final prices $p_0(\underset{\sim}{m})$, $p_1(\underset{\sim}{m})$ do not satisfy (3.1), in which case trade will not occur. In view of this, one party may try to threaten the other. For example, the buyer may send a letter to the seller saying, "If you don't agree to a price substantially below $\hat{p}_1$, I won't buy from you". Such a threat is not credible, however, since the seller knows that if he ignores the letter and, come date 2, supplies the good, it will be accepted. So as long as we include the requirement of perfection or credibility, $\hat{p}_1$ is the unique Nash equilibrium price in the dotted box.[11]

Consider next the triangle North-East of the dotted box, where

(3.2) $$v \geq c > \hat{p}_1 - \hat{p}_0.$$

The seller can always guarantee himself a net return of $\hat{p}_0$ by sending no messages to the buyer, refusing to deliver at date 2, and denying receipt of

any messages in the event of a dispute. Hence if trade does occur, the buyer must pay the seller at least $\hat{p}_0+c$. Proposition 1 tells us that, in the region where (3.2) holds, the buyer need pay no more. The way the buyer achieves this is as follows. First, he sends no messages to the seller until the last mail day before date 2. Then on this last mail day, he sends the following letter: "I propose that we rescind the old contract, and write a new contract with prices $(\hat{p}_0, \hat{p}_0+c)$. If you agree, sign here and retain". Note that the buyer is offering to raise the price in this new contract -- since $\hat{p}_0+c>\hat{p}_1$ by (3.2).

This proposed new contract has the following effect. The seller will now be prepared to supply the good at date 2 since he knows that, if there is a dispute, he can always produce the new signed contract as evidence and obtain at least the price $\hat{p}_0+c$. Moreover, the seller is unable to obtain more than $(\hat{p}_0+c)$ because the buyer can always wait for the seller to produce the new contract and deny receipt of any messages under the old contract which might raise the price above $(\hat{p}_0+c)$. Note that at the unrevised prices $(\hat{p}_0, \hat{p}_1)$, the seller would not agree to supply the good since his net return $\hat{p}_1-c$ is below $\hat{p}_0$.

At first sight it may seem odd that the buyer is able to get all the gains from trade under (3.2) (i.e. the seller is indifferent between not trading under the old contract and trading under the new one). Why can't the seller offer a similar "take it or leave it" new contract at prices $(\hat{p}_0, \hat{p}_0+v)$, thus giving himself all the surplus? The answer is that the buyer has no incentive to sign such a contract. It is better for him, in the event of a dispute, to rely on the old contract, which gives him a price of $\hat{p}_1<\hat{p}_0+v$

(by (3.2)). In other words, the asymmetry between the buyer and seller comes from the fact that, when (3.2) holds, the buyer prefers to take the good than not at the unrevised prices $(\hat{p}_0, \hat{p}_1)$, while the seller does not. So it is the seller who wants, and hence will sign, a new contract, and this gives the buyer the power to dictate terms in the "take it or leave it" contract he offers on the last day before date 2.

In the triangle South-West of the dotted box, where

(3.3) $$\hat{p}_1 - \hat{p}_0 > v \geq c,$$

the asymmetry works the other way. Now the seller has all the power. On the "last day", he offers the new contract $(\hat{p}_0, \hat{p}_0 + v)$ and the buyer is then prepared to accept the good, knowing that he will only have to pay $\hat{p}_0 + v < \hat{p}_1$. The seller can dictate terms here because he is happy to trade under the old contract, while the buyer wants a new contract.

We have now established Proposition 1. Two special cases of the proposition are worth noting. The first is where $k < \underline{v}$ in Figure 2. Then the trading price is $\hat{p}_0 + c$, and the buyer has all the power over the whole region $v \geq c$. The second is where $k \geq \bar{c}$, in which case the price is $\hat{p}_0 + v$, and the seller has all the power over the whole region $v \geq c$.

<u>Remark</u>: The way ex-post surplus is divided is very different from that found in the literature on noncooperative approaches to bargaining (see, for example, Rubinstein (1981)). It might be thought that the result that one party gets all the gains from trade in one sub-region and the other party gets all the gains in the other sub-region is sensitive to our assumption

that trade must occur at date 2 or not at all. This is not so, however. Suppose, for example, that trade can occur up to T "days" after date 2, but that the value v and cost c are discounted at a constant rated $\delta<1$ (and hence the surplus (v-c) shrinks at this rate). The above argument shows that, on the last day, if trade has not yet occurred, the buyer will get all the gains from trade if (3.2) holds (and the seller will if (3.3) holds). But this means that on the penultimate day, the buyer can and will offer the contract which gives him all the gains, and the seller may as well sign this contract and trade then. (Again a new contract offered by the seller will be ignored by the buyer.) Carrying this argument back to the first day, we see that in equilibrium trade will take place at the first opportunity after date 2 at price $\hat{p}_0+c$ if (3.2) holds (and at price $\hat{p}_0+v$ if (3.3) holds). In fact, this same argument generalizes to the case $T=\infty$.

The results of this section can be summarized as follows. When messages cannot be verified, the ability of the buyer and seller to limit the way contractual revisions are made in the future is very small. In fact, the buyer and seller are limited to specifying $\hat{p}_0$, $\hat{p}_1$; once this is done, prices in all states (v,c) are determined according to Figure 2. Given that $\hat{p}_0$ is determined in the ex-ante market for contracts, this means that the parties have a single degree of freedom, $\hat{p}_1-\hat{p}_0\equiv k$. In the next section, we shall see that the parties have many more degrees of freedom if messages between dates 1 and 2 can be verified.

4. CASE (B): SENDING A MESSAGE CAN BE VERIFIED

A major difference between Cases A and B is that, when messages can be verified, the date 0 contract can force each party to send one or more messages from a prescribed set. That is, suppose it is ex-ante desirable for the buyer to send one of the messages $b_1, b_2, b_3 \ldots b_m$ and the seller to send one of the messages $s_1, s_2, s_3 \ldots s_n$ between dates 1 and 2. Then, in Case (B), this can be ensured by a provision which says that the buyer (resp. seller) must pay the seller (resp. buyer) a large sum if he sends a message other than $b_1, b_2, b_3 \ldots$ (resp. $s_1, s_2, s_3 \ldots$) or doesn't send any message at all.

For reasons which will become clear shortly, it is convenient to consider the message game in normal form. As above, let the messages -- or strategies -- of the buyer and seller be $b_1, \ldots, b_m$, $s_1, \ldots, s_n$, respectively.

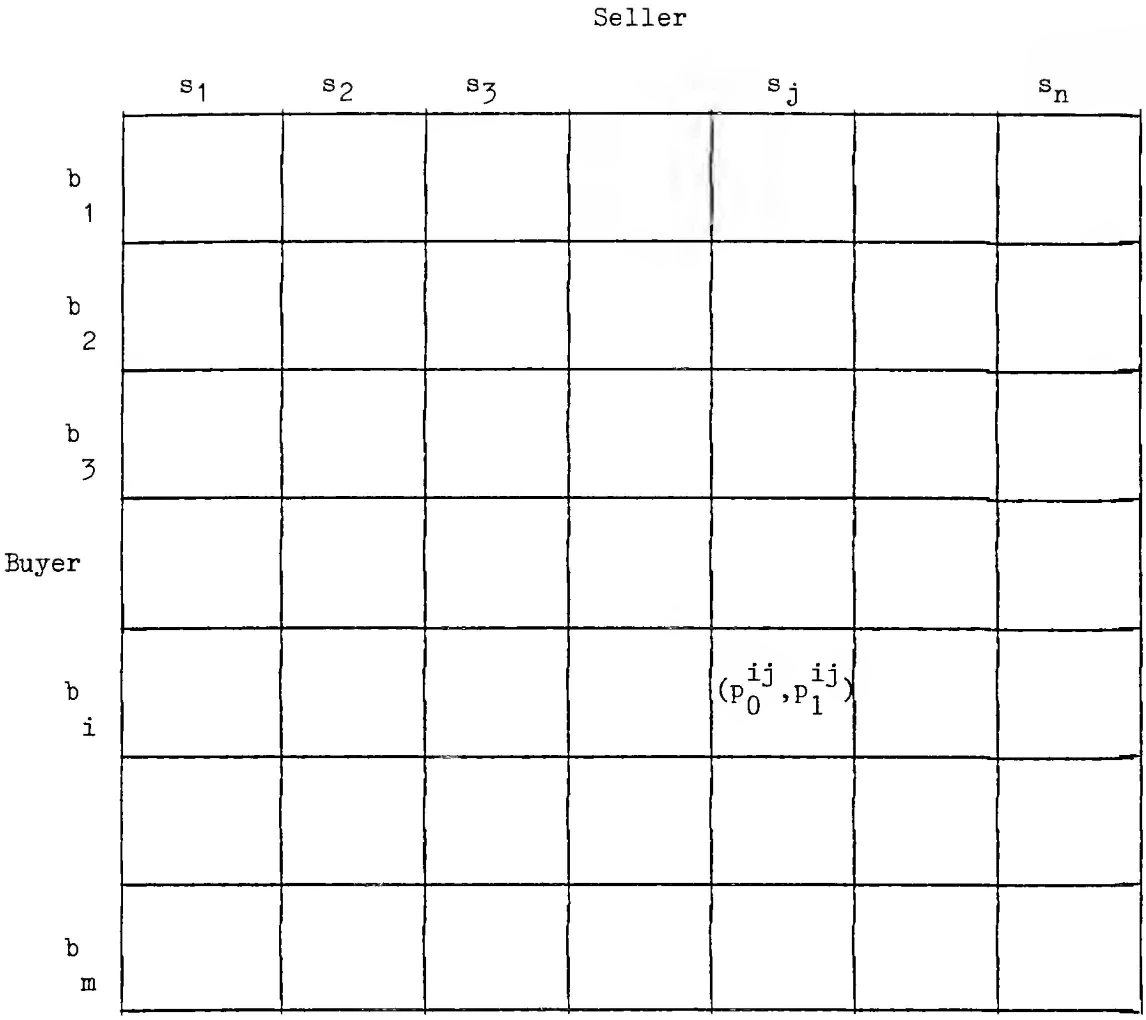

Figure 3

Any pair of messages (b_i, s_j) leads to "revised" contract prices, denoted by (p_0^{ij}, p_1^{ij}), as in Figure 3. That is, if the buyer sends the message b_i and the seller sends the message s_j, the resulting price will be p_1^{ij} if they trade at date 2 and p_0^{ij} if they don't. The messages $b_1, . . ., b_m$ and $s_1, . . ., s_n$, and the mapping from messages to prices, given by the pairs (p_0^{ij}, p_1^{ij}), are choice variables in the date 0 contract.

Although (p_0^{ij}, p_1^{ij}) are revised contract prices, they may not be final prices. The reason is the following. Suppose $v \geq c$, the buyer sends b_i and the seller sends s_j, but $p_1^{ij} - p_0^{ij}$ does not lie between v and c. Then although trade is mutually beneficial, it will not take place under the revised contract. However, it will then be in the interest of the two parties to rescind the revised contract and write a new contract which enables trade to occur. We suppose that this happens in exactly the same way as in Section 3. That is, if $v \geq c$ the final trading price will be:

$$(4.1) \quad p_1^{ij}(v,c) = \begin{cases} p_1^{ij} & \text{if } v \geq p_1^{ij} - p_0^{ij} \geq c, \\ p_0^{ij} + c & \text{if } v \geq c > p_1^{ij} - p_0^{ij}, \\ p_0^{ij} + v & \text{if } p_1^{ij} - p_0^{ij} > v \geq c. \end{cases}$$

On the other hand, if $v < c$, trade will not occur and the price will be p_0^{ij}.

We see then that the possibility that the contract can be renegotiated has an important implication.[12/] The date 0 contract cannot make the "revised" trading price depend directly on v,c (since v,c are not publically observable), but only indirectly via the messages b_i, s_j sent. However, it is clear from (4.1) that renegotiation can lead to a final trading price which depends directly on (v,c). Note that this is not true of the no trade price, p_0^{ij}, which rules if $v < c$, and which depends only on s_i, b_j.

Let us return to the game, illustrated in Figure 3. Suppose first that $v \geq c$. Then it follows from the above argument that, <u>whatever</u> messages b_i, s_j

are sent, trade will occur. This means that the buyer and seller are playing a zero sum game where the payoff, $p_1^{ij}(v,c)$, defined in (4.1), is the amount the seller receives from the buyer (this payoff ignores the buyer's value v and the seller's cost c). Let $p_1^*(v,c)$ be the value of this game, defined by

$$(4.2) \quad p_1^*(v,c) = \underset{\pi}{\text{Min}}\ \underset{\rho}{\text{Max}} \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_i\, \rho_j\, p_1^{ij}(v,c),$$

where $\pi \in \{\pi | \pi \geq 0, \sum_{i=1}^{m} \pi_i = 1\}$, $\rho \in \{\rho | \rho \geq 0, \sum_{j=1}^{n} \rho_j = 1\}$.

Then, by the well-known saddle point property (see, e.g., Luce and Raiffa (1958)), all Nash equilibria of this game (some of which may involve mixed strategies) give the seller an expected payoff of $p_1^*(v,c)$ and the buyer an expected payoff of $-p_1^*(v,c)$. Among other things, this means that, although there may be multiple Nash equilibria, they are equivalent from the point of view of the buyer and seller, i.e. they lead to the same expected payoffs.

If $v < c$, the game is again zero sum, where this time the payoff is p_0^{ij}. We denote the value of this game by

$$(4.3) \quad p_0^* = \underset{\pi}{\text{Min}}\ \underset{\rho}{\text{Max}} \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_i\, \rho_j\, p_0^{ij},$$

where π, ρ have the same domains as above. p_0^* is the expected amount the buyer pays the seller in the event of no trade. Again, while there may be multiple equilibria, they are equivalent for both the buyer and seller.

The fact that the buyer and seller play a zero sum game, both when $v \geq c$ and when $v < c$, justifies our decision to analyze the game in normal form. In particular, since all Nash equilibria of a zero sum game are perfect,

there is no "loss of information" in moving from the extensive form of the game to the normal form.

As we have noted, the task facing the buyer and seller at date 0 is to design an appropriate game, i.e. appropriate messages (or strategies) $b_1, \ldots, b_m, s_1, \ldots, s_n$ and payoffs (p_0^{ij}, p_1^{ij}), in the knowledge that the value of this game will be given by $p_1^*(v,c)$ or p_0^*, depending on whether $v \geq c$ or $v < c$. While p_0^* is a constant, $p_1^*(v,c)$ varies with v,c, and the next question to ask is how. This question is fortunately not difficult to answer, given the zero-sum property of the game.

<u>Proposition 2</u>: Let $p_1^*(v,c)$ and p_0^*, defined in (4.2) and (4.3), be the values of the above game, respectively when $v \geq c$ and when $v < c$. Then

(1) For all $v \geq c$, $p_1^*(v,c)$ is nondecreasing in v and c

(2) If $v' \geq c'$, $v \geq c$, and $p_1^*(v',c') > p_1^*(v,c)$, then

$$p_1^*(v',c') - p_1^*(v,c) \leq \text{Max}\{v' - v,\ c' - c\}$$

(3) For all $v \geq c$, $p_1^*(v,c) - v \leq p_0^* \leq p_1^*(v,c) - c$.

The first and third parts of proposition 2 are not surprising. Part one tells us that the price the buyer must pay for the good cannot fall if the seller's cost rises or if the buyer's valuation rises. The third part says that neither the buyer nor the seller can be worse off trading than not. The second part is a bit less intuitive, but it says, among other things, that if v and c both rise by α, p_1^* rises by no more than α.

Proof:[13] To prove Part 1, note that the final price contingent on messages b_i and s_i being sent, given by $p_1^{ij}(v,c)$ in (4.1), is nondecreasing in v and c. Hence, if $v' \geq v$ and $c' \geq c$,

$$(4.4) \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_i \rho_j p_1^{ij}(v',c') \geq \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_i \rho_j p_1^{ij}(v,c) \quad \text{for all } \pi,\rho.$$

From this it follows that

$$(4.5) \quad \operatorname{Max}_{\rho} \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_i \rho_j p_1^{ij}(v',c') \geq \operatorname{Max}_{\rho} \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_i \rho_j p_1^{ij}(v,c) \quad \text{for all } \pi,$$

and hence that

$$(4.6) \quad \operatorname{Min}_{\pi} \operatorname{Max}_{\rho} \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_i \rho_j p_1^{ij}(v',c') \geq \operatorname{Min}_{\pi} \operatorname{Max}_{\rho} \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_i \rho_j p_1^{ij}(v,c).$$

This establishes Part 1.

In Part 2, set $\alpha = \operatorname{Max}\{v' - v,\ c' - c\}$. If $p_1^*(v',c') > p_1^*(v,c)$, then from Part 1, $\alpha > 0$. If $v' - v = c' - c = \alpha$, it follows from (4.1) that

$$(4.7) \quad p_1^{ij}(v',c') - p_1^{ij}(v,c) \leq \alpha \qquad \text{for all } i \text{ and } j.$$

Hence, again using Part 1, we see that (4.7) holds if either $v' - v < c' - c = \alpha$ or

$c' - c < v' - v = \alpha$. Part 2 is then proved by applying the argument of (4.5) - (4.6) to (4.7).

To prove Part 3, observe that, from (4.1),

$$(4.8) \quad p_1^{ij}(v,c) - v \leq p_0^{ij} \leq p_1^{ij}(v,c) - c \quad \text{for all } i \text{ and } j.$$

Now apply the argument of (4.5) - (4.6) to (4.8). Q.E.D.

Proposition 2 states <u>necessary</u> conditions for $p_1^*(v,c)$ and p_0^* to satisfy

if they are to be generated by a message game. The next proposition tells us that these conditions are <u>sufficient</u>.

<u>Proposition 3</u>: Suppose $\tilde{p}_1(v,c)$ and $\tilde{p}_0$ satisfy

(1) For all $v \geq c$, $\tilde{p}_1(v,c)$ is nondecreasing in v and c

(2) If $v' \geq c'$, $v \geq c$, and $\tilde{p}_1(v',c') > \tilde{p}_1(v,c)$ then

$$\tilde{p}_1(v',c') - \tilde{p}_1(v,c) \leq \text{Max}\ \{ v' - v,\ c' - c \}$$

(3) For all $v \geq c$, $\tilde{p}_1(v,c) - v \leq \tilde{p}_0 \leq \tilde{p}_1(v,c) - c$.

[If v is never less than c (so that $\tilde{p}_0$ is not defined) then condition (3) should be prefaced by "There exists a $\tilde{p}_0$ such that ..."]

Then there exist messages $b_1, \ldots, b_m, s_1, \ldots, s_n$ and payoffs $(p_0{}^{ij}, p_1{}^{ij})$ such that $p_1^*(v,c)$, defined in (4.2), equals $\tilde{p}_1(v,c)$ for all $v \geq c$ and p_0^*, defined in (4.3), equals $\tilde{p}_0$.

<u>Proof</u>: See Appendix B.

The proof of Proposition 3 is very much in the spirit of Maskin's work (1983) on the implementation of welfare optima as Nash equilibria. A game is constructed in which each party announces a pair (v_i, c_j), which can be interpreted as its version of the true values (v,c). The Nash equilibrium of the game is such that each party wants to tell the truth. Two differences from Maskin's work can be noted. First, the game we consider is a two stage one where in the first stage a price pair (p_0, p_1) is determined, and in the second stage trade occurs only if both parties want it to. In Maskin's work,

the game analyzed has a single stage, and a "voluntary trade" requirement is not included (although it appears that it could be). Secondly, Maskin does not allow for renegotiation. This means among other things that if one party does not tell the truth, trade may not occur even when $v > c$; in other words, the game he considers is not zero sum. (It turns out that in the game constructed to prove Proposition 3, renegotiation does not occur in equilibrium; however, the possibility of it influences out of equilibrium behavior. In particular, it prevents the existence of Nash equilibria other than the truth-telling one.)[14]

5. APPLICATIONS

We have characterized the ex-post divisions of the surplus that the parties can achieve, both for the case where messages cannot be verified and for the case where they can be. We now consider what implications our results have for the form of the optimal second-best contract.

As we noted above, there are two reasons why the ex-post division of surplus matters. First, the parties may be risk-averse; secondly, they may take actions which affect the size of this surplus. To simplify matters, we consider these two factors separately, although both would be present in a general model.

Consider first the case where the parties are risk-averse, i.e. the buyer's utility is $B(vq - p)$ and the seller's utility is $S(p - cq)$, where $q=0$

or 1 is the level of trade. We suppose that B,S are defined respectively on intervals (I_B, ∞), (I_S, ∞) of the real line, where $\lim_{I \to I_B} B(I) = \lim_{I \to I_S} S(I) = -\infty$, and $B' > 0$, $B'' \leq 0$, $S' > 0$, $S'' \leq 0$.

A first-best (contingent) contract is characterized as follows:

(5.1) $q(v,c) = 1 \iff v \geq c$;

(5.2) $p(v,c) = p_0$ when $v < c$;

(5.3) $\dfrac{B'(v - p(v,c))}{B'(-p_0)} = \dfrac{S'(p(v,c) - c)}{S'(p_0)}$ for all $v \geq c$;

(5.4) $\int S(p(v,c)\, q\, (v,c) + p_0(1-q(v,c)) - cq(v,c))dF(v,c) = \bar{U}$.

Here $p(v,c)$ is the payment from the buyer to the seller in state (v,c) and F is the distribution function of (v,c). (5.2) is the Borch optimal risk-sharing condition and (5.3) says that the seller's ex-ante expected utility is $\bar{U}$, determined in the market for contracts. (5.2) tells us that the no-trade price is a constant, p_0, while it follows from (5.3) that

(5.5) $v - p(v,c) = \phi(v-c)$,

$p(v,c) - c = v-c-\phi(v-c)$,

where ϕ is a function satisfying $0 \leq \phi' \leq 1$. That is, each party's net payoff is a function only of total surplus Max $(v-c, 0)$. Together (5.2) and (5.5) imply that (1) - (3) of Proposition 3 are satisfied, where $\tilde{p}_0 \equiv p_0$, $\tilde{p}_1(v,c) \equiv p(v,c)$. It follows that, with verifiable messages, the first-best can be achieved even though contingent contracts cannot be written directly.

When messages cannot be verified, the situation is very different. From Proposition 1, the optimal contract solves:

$$\text{Max} \Big[\int_{v \geq k \geq c} B(v-k-p_0)\, dF(v,c) + \int_{v \geq c > k} B(v-c-p_0)\, dF(v,c)$$

$$+ \int_{k > v \geq c} B(-p_0)\, dF(v,c) + \int_{v<c} B(-p_0) dF(v,c) \Big]$$

$$\text{S.T.} \int_{v \geq k \geq c} S(p_0 + k - c)\, dF(v,c) + \int_{v \geq c > k} S(p_0)\, dF(v,c)$$

$$+ \int_{k > v \geq c} S(p_0 + v - c)\, dF(v,c) + \int_{v<c} S(p_0) dF(v,c) \geq \bar{U}.$$

It is clear from Figure 2 that only quite crude divisions of the surplus are possible, and price functions of the form (5.5) cannot generally be implemented. In some cases, this may not matter. For example, if the buyer is risk-neutral, the first-best can be achieved by setting $k < \underline{c}$, since then the trading price is $p_0 + c$, which means that the seller is fully insured. Similarly, if the seller is risk neutral, the first-best can be achieved by setting $k > \bar{v}$; this yields the trading price $p_0 + v$, and the buyer is fully insured. In general, however, the division of the surplus indicated in Figure 2 will be quite undesirable from a risk-sharing point of view, and the loss from being in the second-best when messages cannot be verified is likely to be quite large.

Consider now the case where the parties are risk-neutral, but take actions to increase the surplus $(v-c)$. Let the buyer's action be β and the seller's action σ, and suppose these are taken just after the date 0 contract is signed (we can think of β and σ as being specific investments). We assume that each party's action increases only his own payoff, i.e. β enhances v and σ diminishes c. To simplify we suppose that v,c are independent, and we write their distribution functions as

$$F_b(v,\beta),\ F_s(c,\sigma).$$

If the actions β, σ were publically observable, their existence would cause no new problems. The contract would simply specify the optimal β and σ, with each party having to pay the other party a large sum if he did not carry out the appropriate action. We shall suppose, however, that β, σ are "private" variables, which are observable only to the party carrying them out (they correspond to effort, say). As a result, the contract cannot mention β, σ explicitly, but can only gear payments to the resulting realizations of v and c.

Before we analyze the second-best, it is again useful to consider the first-best -- where β, σ can be specified and prices can be made contingent on (v,c). The optimal first-best contract solves:

$$(5.6)\qquad \underset{\beta,\sigma}{\text{Max}} \int_{v\geq c} (v-c)\ dF_b(v,\beta)\ dF_s(c,\sigma) - h_b(\beta) - h_s(\sigma),$$

where h_b, h_s are the costs (in monetary) terms to the buyer and seller of the actions β, σ. That is, because of the risk neutrality of the buyer and seller, β, σ should be chosen to maximize expected surplus (this surplus will then be divided up so that the seller receives $\bar{U}$ by means of a date 0 side payment). Let the solution of this problem be β^*, σ^*.

In fact it turns out that even though β^*, σ^* cannot be specified in the contract, the first-best can still be implemented as long as a contingent contract can be written on v and c. This may be achieved as follows: Define $p(v,c)$, the amount the buyer pays the seller in state (v,c), by

$$(5.7)\quad p(v,c) = \int_{v \geq \tilde{c}} \tilde{c}\, dF_s(\tilde{c}, \sigma^*) + \int_{\tilde{v} \geq c} \tilde{v}\, dF_b(\tilde{v}, \beta^*).$$

That is, the buyer pays average cost conditional on cost being less than the realized value multiplied by the probability of this event plus average benefit conditional on benefit exceeding realized cost multiplied by the probability of this event -- both evaluated with respect to the optimal actions, β^*, σ^*. Then one contract which achieves first-best is:

$$(5.8)\quad \begin{array}{lll} \text{If } v \geq c: & p_1(v,c) = p(v,c) & \\ & \text{and } p_0(v,c) = p(v,c) - k, & \text{where } k \text{ is any number in } [c,v] \\ \text{If } v < c: & p_0(v,c) = p(v,c) & \\ & \text{and } p_1(v,c) \text{ is arbitrary.} & \end{array}$$

This contract leads to efficient trading: $q=1$ iff $v \geq c$, and it is not difficult to show that it induces efficient actions. Note that there are also generally other schemes which implement first-best -- we do not have uniqueness here.

Unfortunately, this pricing scheme does not generally satisfy the conditions of Proposition 3 since, among other things, $p(v,c)$ is not constant over states $v < c$. (This is also true of other schemes which implement the first-best.) This is not surprising. In order to implement β^* and σ^*, it is necessary to reward (penalize) the buyer when v is high (low) and the seller when c is low (high), and over the no-trade region this is achieved via variations in $p(v,c)$.

Thus in general it is impossible to achieve the first-best in the action-risk neutrality case using a verifiable message scheme, and hence a fortiori using a non-verifiable message scheme. There are some exceptions to this, and these are grouped together in the next proposition.

Proposition 4: In the action-risk neutrality case, the first-best can be achieved using a non-verifiable message scheme if any one of the following conditions holds:

(1) $v \geq c$ with probability 1 for all β, σ; (2) $F_b(v,\beta)$ is independent of β; (3) $F_s(c,\sigma)$ is independent of σ.

To establish (1), note that, if $v \geq c$ always, $p(v,c)$ is a constant in (5.7). Also $\underline{v} \geq \bar{c}$ (since v,c are independent). Hence simply choose k such that $\underline{v} \geq k \geq \bar{c}$ and let the contract prices be $(p_0,\ p_0 + k)$, where p_0 is such that the seller receives an expected utility of $\bar{U}$.

In case (2), the buyer's action is irrelevant. The first-best is now achieved by choosing $k > \bar{v}$ so that the buyer never wants to trade under the original contract. This means that when the contract is revised at date 1, the trading price will be $p_0 + v$, and hence the seller's objective function is $(v-c)$; this ensures that he takes an efficient action. Case (3) works the other way round: now $k < \underline{c}$, so that the revised price is $p_0 + c$, and the buyer takes an efficient action.

(1) - (3) are very strong conditions. In general, if $v < c$ with positive probability and both the buyer's and seller's actions matter, the

first-best cannot be achieved. This raises the question of what is the form of the optimal second-best contract, i.e. the contract which maximizes (5.6) subject to (p_0, $p_1(v,c)$) satisfying (1) - (3) of Proposition 2 (in the case of verifiable messages) and (1)-(4) of Proposition 1 (in the case of non-verifiable messages). One case where we have been able to answer this question is:

<u>Proposition 5</u>: In the action-risk neutrality case, if β and σ can be scaled so that they both lie in $[0,1]$, and

(1) for each β in $(0,1)$, the (non-degenerate) support of $F_b(v,\beta)$ is

$$\{\, \underline{v} = v_1 < \dots < v_i < \dots < v_I = \bar{v} \,\} \qquad (I \geq 2)$$

and the probability of v_i is

$$\pi_i(\beta) = \beta\pi_i^+ + (1-\beta)\pi_i^-$$

where $\underset{\sim}{\pi}^+$ and $\underset{\sim}{\pi}^-$ are probability distributions over $\{v_1, \dots, v_I\}$ and π_i^+/π_i^- is increasing in i

(2) for each σ in $(0,1)$, the (non-degenerate) support of $F_s(c,\sigma)$ is

$$\{\bar{c} = c_1 > \dots > c_j > \dots > c_J = \underline{c}\,\} \qquad (J \geq 2)$$

and the probability of c_j is

$$\rho_j(\sigma) = \sigma\rho_j^+ + (1-\sigma)\rho_j^-$$

where $\underset{\sim}{\rho}^+$ and $\underset{\sim}{\rho}^-$ are probability distributions over $\{c_1, \dots, c_J\}$ and ρ_j^+/ρ_j^- is increasing in j

(3) $h_b(\cdot)$ and $h_s(\cdot)$ are convex and increasing in $[0,1]$, with

$$\lim_{\beta\to 0} h_b'(\beta) = \lim_{\sigma\to 0} h_s'(\sigma) = 0$$

and $$\lim_{\beta\to 1} h_b'(\beta) = \lim_{\sigma\to 1} h_s'(\sigma) = \infty$$

(4) $\underline{v} < \bar{c}$ and $\bar{v} > \underline{c}$

then, even if messages are verifiable, the second-best can be achieved using a non-verifiable message scheme. Also the second-best actions β and σ are both strictly less than their respective first-best levels β^* and σ^* (assumed unique).[15]

Remark. If (i) the first part of condition (4) did not hold (i.e. if $\underline{v}>\bar{c}$),or

(ii) the distribution $F_b(v,\beta)$ were degenerate (i.e. I = 1), or

(iii) the distribution $F_s(c,\sigma)$ were degenerate (i.e. J = 1),

then we know from Parts (1), (2), and (3) respectively of Proposition 4 that the first-best can be achieved using a non-verifiable message scheme.

Conditions (1) - (3), then, are sufficient to ensure that there is some simple two-price contract p_0, p_1 -- as in Proposition 1 -- which performs just as well a more sophisticated contract which uses verifiable messages (of course if messages cannot be verified then the contract would have to be simple anyway). Condition (3) ensures a unique, interior solution for β and σ. Conditions (1) and (2) are the important ones. They amount to a combination of the Spanning Condition and the (strict) Monotone Likelihood Ratio Condition discussed in Grossman and Hart (1983, pages 23 and 25).

The rough intuition behind Proposition 5 is as follows. Suppose messages can be verified. A second-best contract will induce the buyer and

seller to take actions as close to β^* and σ^* as possible, given the constraints on what divisions of surplus can be implemented. One constraint is that the trading price $p_1^*(v,c)$ has to be non-decreasing in v and c (Proposition 2(1)). Consider the buyer. From condition (1) in Proposition 5, the larger (smaller) the buyer's surplus in the high (low)v_i states, the larger will be β. So, if the second-best β is less than β^*, then $p_1^*(v,c)$ should increase with v as little as possible. A similar argument for the seller, using condition (2), suggests that if the second-best σ is less than σ^*, then $p_1^*(v,c)$ should increase with c as little as possible. But there are other constraints on $p_1^*(v,c)$; in particular trade has to be voluntary, i.e.

$$p_0^* + c \leq p_1^*(v,c) \leq p_0^* + v \qquad \text{(Proposition 2(3))}$$

The best "compromise" set of trading prices has the form given in figure 2, and this can be achieved without verifiable messages.[16]

To summarise the results of this section: we have examined two cases in which the ex post division of surplus matters, first where the parties are risk averse, and second where the parties are risk neutral but take actions which affect the size of the surplus. From section 3 we know the form the trading prices must take if messages cannot be verified, and in both our cases the first-best can rarely be achieved with such a crude division of surplus. The question is, what difference does it make if messages can be verified and the trading prices can take the more subtle form given in section 4? In the risk aversion case, it turns out that the first-best can now be implemented. On the other hand, for a particular example of the action-risk neutrality case, being able to verify messages makes no difference.

6. Conclusions

It is usually argued that a contract is important in facilitating trade between two parties who have a long-term trading relationship involving large specific investments (see, e.g., Williamson (1979)). This is because ex-post, competition will have little impact on the terms of the trading relationship and so these must be governed instead by contractual provision. A major problem facing the drafters of the contract is to anticipate and deal appropriately with the many contingencies which may arise during the course of what may be a very long relationship (in the coal industry, some contracts involving coal mines and electricity generating plants last more than thirty years; see Joskow (1984)). In fact, since it does not pay to plan for every conceivable eventuality, contracts will typically contain large gaps.

In this paper, we have considered how these gaps might be filled in during the course of the trading relationship. We have studied a situation where the gaps are due to the inability of the parties ex-ante to describe the objective events which will ex-post determine the state of the world (another interpretation is that the parties are initially unaware of the relationship between these events and the state). The parties can make up for this to some extent by building into the contract a mechanism for revising the terms of trade as each party receives information about benefits and costs. We have studied the design of an optimal mechanism of this type under two different assumptions abut the communication mechanism at the parties' disposal.

A natural question to ask is whether mechanisms of the sort that we have described are found in practice. It is very common for long-term contracts to contain formulae linking future terms of trade to some objective industry price or cost index, or to actual cost through a cost plus arrangement. Our mechanism is rather different, however, in that it involves one or both parties having a direct influence over the terms of trade (the mechanism could, of course, easily be supplemented by the use of external indexes or cost plus arrangements). It is worth noting that, in his interesting study of long-term contracts involving coal suppliers and electricity generating plants, Joskow (1984) discusses a case of a contract which gave the coal supplier an option to switch from an indexed arrangement to a cost plus arrangement on six months notice. This is a special case of the mechanism we consider (in general, both parties will have some choice over the price schedule), although it is also consistent with certain asymmetric information mechanisms (see, e.g., Riordan (1984)).

One striking conclusion of our analysis is because the parties can ex post rescind the original contract and negotiate a new one, severe limits are placed on the price revision schemes which are feasible ex ante. It may be noted that in equilibrium no renegotiation is ever required (we have already noted this in the verifiable message case; in the nonverifiable case, the Figure 2 outcome can be achieved by specifying the prices $\hat{p}_0$, $\hat{p}_0+k$ and giving the buyer the power to raise prices and the seller the power to lower them). Our supposition is that this is connected to the assumption of unbounded rationality. Since the parties have unlimited ability to conceive of all the possible benefit-cost situations -- that is, (v,c) pairs -- any renegotiation can be

anticipated and built into the revision process embodied in the original contract.

In order to understand renegotiation as an equilibrium phenomenon, it would seem therefore that we must drop the assumption of unbounded rationality. It goes without saying that this is a vital -- if forbiddingly difficult -- topic for future research.

Footnotes

1. The courts are assumed not to know how v and c depend on ω. We are thus using the term state of the world rather loosely here since it does not include the values of v and c; by the state we mean those factors or events which are publically observable.

2. Revisions have also been studied by Rogerson (1984) and Shavell (1984).

3. We suppose that this revision and/or renegotiation is costless, which is clearly an extreme assumption. The opposite assumption -- that they are prohibitively costly -- which is also extreme, was made in Grossman and Hart (1984).

4. We confine attention to a contract between a separately owned buyer and seller; that is, we do not consider the possibility that the parties might resolve their contractual problems by vertically integrating. For an analysis of this, see Grossman and Hart (1984).

5. We adopt the convention that q = 1 if v = c.

6. In other words, v and c are observable, but not verifiable; as in Holmstrom (1982) or Bull (1983).

7. In order to justify the assumption that v,c are not observable to a court, we must suppose that these are private benefits and costs which accrue directly to the managers of the two firms -- i.e., like effort, they don't show up in the accounts. More generally, actual benefits and costs might be observable, but the relationship between these observable variables and the unobservable effort levels of managers might be uncertain. It seems likely that our analysis could be extended to this case.

8. $(p_0(\underset{\sim}{m}), p_1(\underset{\sim}{m}))$ can be thought of as a nonlinear price schedule. Alternatively, $p_0(\underset{\sim}{m})$ can be thought of as the damages the buyer pays the seller (which might be negative) if "breach" occurs (in legal terms, these are "liquidated" damages).

9. It is worth noting that the extension of this rule to the case where q takes on more than two values is by no means obvious.

10. The telegraph company is a third party, and hence the same factors which make the inclusion of any third party in the contract problematical may be relevant here (see Appendix A). Case B may, however, apply in the absence of a third party if, say, messages are transmitted by telephone and can be recorded (and the recordings cannot be meddled with). Note that registered mail does not satisfy the conditions of Case B since, although it may be established that a message was sent, there is no record of the contents.

11. There is a second sort of threat which is potentially more powerful. The buyer, say, could send the following two letters to the seller at

the same time. The first letter says: "If we ever trade under the old contract, I agree to pay you a large sum over and above $\hat{p}_1$." The second letter says: "I propose that we rescind the old contract and sign a new contract which specifies prices ($\hat{p}_0$, $\hat{p}_0$+c). Please write back confirming that you agree to this." (The letters are sent so that the seller has enough time to respond before date 2.) The first letter poses a serious problem for the seller. If he hangs on to it, the buyer knows that if q=1, the seller will produce it in the event of a dispute in order to obtain a very high price. The seller, knowing this, realizes that the buyer will not accept the good at date 2. Hence the seller may as well sign the new contract, which gives him a price less than $\hat{p}_1$. We implicitly assume that the seller can neutralise this threat in one of two ways: he either returns the letter to the buyer or he writes a new letter saying "I reject the terms of your first letter." In both cases, this has the effect of returning the buyer and seller to the position where trade will occur at price $\hat{p}_1$.

12. The exact way the contract is renegotiated may be more complicated than in Section 3. Suppose, for example, the messages b_i, s_j are sent on the last day, n. Then since there is no time left for renegotiation, the new contract must be exchanged at the same time. Given that it is not yet clear what the prices under the old contract will be or who has the power to dictate the terms of this new contract, one can imagine that each party sends a new contract on day n, proposing prices which are contingent on the message the other party sends on that day. By date 2, it will be clear what the old contract prices are and which of these new contracts has force; the revised prices will then be given by (4.1)

13. We are grateful to Eric Maskin for providing the following argument.

14. It can be shown, both in the nonverifiable and verifiable cases, that the possibility of ex post renegotiation reduces the set of feasible contracts ex ante. In view of this, one might ask whether the parties can constrain themselves not to use the renegotiation option. One possibility is for them to agree that any suggestion by one to the other (through the mail, say) that the old contract should be rescinded should be heavily penalised. This may be difficult to arrange for two reasons. First, certain rescissions and negotiation may be desirable (although we have not modelled this), and it may be difficult to specify in advance which these are. Secondly, the party proposing rescission could take the new contract personally to the other party, parting with it only once it has been signed by this party; the new contract, moreover, could contain a clause waiving the penalty.

15. Underinvestment results have also been established by Grout (1984), Tirole (1985), although in the absence of a date 0 contract.

16. It is of interest to note that the third constraint on $p_1^*(v,c)$ -- given in Proposition 2(2) -- is nowhere binding (with a positive multiplier) in the second-best contract for this action-risk neutrality model.

17. The argument that collusion may threaten three party contracts has also been made by Tirole (1985). Note that the collusion we discuss is rather different from that found in the incentive compatibility literature (see, e.g., Green and Laffont (1979)). There it is typically

supposed that the colluding parties can share private information, an assumption we don't make.

18. Given the assumption that third parties are corruptible, the reader may wonder whether it is reasonable to suppose that the courts are not. One justification is that, given the possibility of appeal, several courts may be involved in judging the case and it may be difficult for one party to bribe them all (in contrast, there is a single designated third party).

19. We are grateful to Steve Matthews for this argument.

References

C. Bull, (1983), "The Existence of Self-Enforcing Implicit Contracts", mimeo, New York University.

J. Dawson, W. Harvey and S. Henderson (1982), Contracts - Cases and Comment, 4th Edition, Foundation Press.

R. Dye (1981), "Costly Contract Contingencies", mimeo, University of Chicago, 1981.

J. Green and J. J. Laffont (1979), "On Coalition Incentive Compatibility", Review of Economic Studies, 243-254.

S. Grossman and O. Hart (1983), "An Analysis of the Principal-Agent Problem", Econometrica, January, 1983.

S. Grossman and O. Hart (1984), "The Costs and Benefits of Ownership: A Theory of Vertical Integration", ICERD discussion paper, London School of Economics

P. Grout (1984), "Investment and Wages in the Absence of Binding Contracts: A Nash Bragaining Approach", Econometrica.

R. Hall and E. Lazear (1984), "The Excess Sensitivity of Layoffs and Quits to Demand", Journal of Labor Economics, Vol. 2, No. 2, April.

B. Holmstrom (1982), "Moral Hazard in Teams," Bell Journal of Economics, Vol. 13, No. 2, Autumn.

P. Joskow (1984), "Vertical Integration and Long Term Contracts: The Case of Coal Burning Electric Generating Plants", MIT working paper, December.

R. Luce and H. Raiffa (1958), "Games and Decisions: Introduction and Critical Survey", John Wiley.

E. Maskin (1984), "The Theory of Implementation in Nash Equilibrium: A Survey", mimeo.

M. Riordan (1984), "Uncertainty, Asymmetric Information and Bilateral Contracts", Review of Economic Studies, January.

W. Rogerson (1984), "Efficient Reliance and Damage Measures for Breach of Contract", Rand Journal of Economics, Spring.

A. Rubinstein (1981), "Perfect Equilibrium in a Bargaining Model", Econometrica, 50.

S. Shavell (1980), "Damage Measures for Breach of Contract", Bell Journal of Economics, II, Autumn.

S. Shavell (1984), "The Design of Contracts and Remedies for Breach", Quarterly Journal of Economics, February.

H. Simon (1981), The Sciences of the Artificial, MIT Press.

J. Tirole (1985), "Procurement and Renegotiation", mimeo, MIT.

M. Weitzman (1981), "Toward a Theory of Contract Types", mimeo, MIT.

O. Williamson (1979), "The Transaction-Cost Economics: The Governance of Contractual Relations", Journal of Law and Economics, 22, October.

Appendix A

We have studied contracts involving only the buyer and seller. At several points, however, we have noted that it may be desirable for the two parties to include a third party in the date 0 contract. This can be seen readily in the action-risk neutrality case. Call the third party T and assume that he is risk neutral (this may be a reasonable assumption if T can diversify by acting as a third party in a large number of independent ventures). Then the first-best can be achieved in the following manner. The contract states that (1) at date 1, the buyer (B) sends T a message announcing his benefit v_a, and the seller (S) sends a message announcing his cost c_a; (2) q=1 if and only if $v_a \geq c_a$; (3) if q=1, B pays T c_a and T pays S v_a, while if q=0, payments are zero.

This "Groves-type" scheme will elicit the truth from B and S at date 1 since neither's payment depends on his announcement. It also ensures efficient actions since B and S's payoffs (gross of effort) are both equal to social surplus (v-c). T makes an expected loss from participating in the contract, but he can be compensated by an appropriate sidepayment at date 0.

While there may be large potential efficiency gains from the inclusion of a third party, various practical problems may prevent these gains from actually being realized. The most serious of these involves the possibility of collusion by two of the parties against the third. For example, in the case described above, there is an incentive for B and T to write a new "side-

contract" just after the initial three party contract is signed. This new side-contract says that all payments made by T to S under the original contract must be matched by payments from B to T and that all payments made by B to T must be returned. This arrangement is equivalent to a merger between B and T, with T's net payment becoming zero in every state, i.e. B buys T out. B's new payment, on the other hand, becomes

$$c_a - c_a + v_a = v_a.$$

Obviously T is indifferent to this merger. B cannot be worse off since he can always choose the same action as without the merger and, given that he is risk neutral, the change in the distribution of returns is of no consequence to him. (We are implicitly assuming that S doesn't observe the writing of the new contract until after he takes his action; otherwise <u>his</u> action might change -- for more on this, see below.) In fact it is easy to show that B will be better off.

Exactly the same argument shows that there is an incentive for S and T to merge.

One way to avoid these mergers, of course, is to prohibit them in the original contract. This may be problematical, however, for two reasons. First, there may be a perfectly legitimate reason for B and T (or S and T) to write certain sorts of new contracts with each other, and it may be difficult to specify in advance which new contracts are allowable and which are not. Secondly, the side-contract may be very complicated, involving subsidiaries of the two companies or intermediaries. For example, B might merge with X

who might merge with Y... who might merge with T. It may be very difficult to give an exhaustive list in the date 0 contract of all illegitimate combinations of such side-arrangements.

If for these reasons, side-contracts cannot be prevented, the above argument shows that the first-best will not be achievable using a Groves scheme in the action-risk neutrality case. In fact the argument establishes more. Consider <u>any</u> three party contract involving B, S and T, where B, S, T are risk neutral. Then, since B and T (or S and T) cannot lose from merging, this contract must be equivalent to a two party contract involving just B and S. In other words, we may as well focus on two party contracts from the beginning.

So far we have assumed that the parties are risk neutral. Is there a more subtle form of side-contracting which would upset a three party contract when, say, both the buyer and seller are risk averse? The following line of argument suggests that there is. But it must be pointed out that this is not a full analysis of the problem because we do not model the side-contracting game explicitly.

Suppose we drop the assumption that q is publically observable. Instead we suppose that (i) the buyer and seller can observe q; (ii) after date 2, each has enough evidence to "prove" to a court what the true q, q* say, was; but (iii) the two parties, if they cooperate, can falsify the evidence to make it look as if a q other than q* occurred. (We are adapting an argument given in Grossman and Hart (1984).)

Let the three party contract state that if messages $\underset{\sim}{m}$ are sent B should pay $p_1^{\,b}(\underset{\sim}{m})$, $p_0^{\,b}(\underset{\sim}{m})$ to T if $q = 1,0$ respectively, and T should pay $p_1^{\,s}(\underset{\sim}{m})$, $p_0^{\,s}(\underset{\sim}{m})$ to S if $q = 1,0$ respectively. To simplify matters we suppose that messages are verifiable -- but a similar argument applies in the nonverifiable case. Then T's net total receipts from B and S are $\lambda_1(\underset{\sim}{m}) = p_1^{\,b}(\underset{\sim}{m}) - p_1^{\,s}(\underset{\sim}{m})$ if $q = 1$, and $\lambda_0(\underset{\sim}{m}) = p_0^{\,b}(\underset{\sim}{m}) - p_0^{\,s}(\underset{\sim}{m})$ if $q = 0$.

Define $\underset{\sim}{m}_{min}$, q_{min} to be any choice of messages $\underset{\sim}{m}$ and trading decision q (= 0 or 1) which minimize $\lambda_q(\underset{\sim}{m})$. And let the minimum value be λ_{min}. Now consider any three party contract in which, for some realized pair (v,c), the <u>equilibrium</u> messages, $\underset{\sim}{m}^*(v,c)$, and trading decision, $q^*(v,c)$, are such that $\lambda_{q^*(v,c)}(\underset{\sim}{m}^*(v,c)) - \lambda_{min} = \Delta\lambda > 0$. B and S can do better by writing a side-contract in which they agree, in the event (v,c), to send messages $\underset{\sim}{m}_{min}$, and (if necessary) forge the trading decision q_{min} as in (iii), and divide up the additional surplus $\Delta\lambda$ between them.

It follows that we may assume without loss of generality that in equilibrium T always receives λ_{min}. But in order to ensure that T breaks even, λ_{min} must therefore be zero.

This does not yet prove that the third party is playing no role in the contract, though. <u>Off</u> the equilibrium path, $\lambda_q(\underset{\sim}{m})$ may be positive. That is, an equilibrium may be sustained via the threat of having (on balance) to pay

T something. But there would then be scope for B (or S) to write a side-contract with T in the way we described earlier, so as to effect a merger. B cannot be worse off from having merged with T, because he could always mimic his previous action, messages, and trading strategy -- leaving his risk position unaffected since T plays no role in equilibrium. It is easy to see that B could do better -- or, if not, then S could.

In sum, if the three party contract is to be invulnerable to side-contracting, then it cannot involve net payments to T: $\lambda_q(\underset{\sim}{m})$ must be identically zero. But then the contract is in effect a two party one involving only B and S.[17/] [18/]

Appendix B

B1. Proof of Proposition 3

Take all the distinct pairs (v,c) which have a positive joint probability, and for which trading is efficient, i.e. $v \geq c$. Suppose there are $t \geq 1$ such pairs. Then number them

$$w_1 = (v_1, c_1) , w_2 = (v_2, c_2), \ldots , w_t = (v_t, c_t),$$

where w is just a shorthand for (v,c). Finally, let w_{t+1} stand for all (v,c) pairs (if any) which have a positive joint probability and $v < c$.

We choose the payoffs of the message game in such a way that in equilibrium both parties want to tell the truth. Note that we cannot punish the parties for "disagreeing" about (v,c) since, in the absence of a third party, one party's punishment is another's reward.

The construction of the game is illustrated in figure 4.

Seller

	w_1	w_2	w_3		w_{t+1}
w_1	$(\tilde{P}_1(\omega_1)-v_1, \tilde{P}_1(\omega_1))$				$(\tilde{P}_1(\omega_1)-c_1, \tilde{P}_1(\omega_1))$
w_2		$(\tilde{P}_1(\omega_2)-v_2, \tilde{P}_1(\omega_2))$			$(\tilde{P}_1(\omega_2)-c_2, \tilde{P}_1(\omega_2))$
w_3			$(\tilde{P}_1(\omega_3)-v_3, \tilde{P}_1(\omega_3))$		$(\tilde{P}_1(\omega_3)-c_3, \tilde{P}_1(\omega_3))$
Buyer					
w_{t+1}	$(\tilde{P}_1(\omega_1)-v_1, \tilde{P}_1(\omega_1))$	$(\tilde{P}_1(\omega_2)-v_2, \tilde{P}_1(\omega_2))$	$(\tilde{P}_1(\omega_3)-v_3, \tilde{P}_1(\omega_3))$		$(\tilde{P}_0, \tilde{P}_0)$

Figure 4

The diagonal elements, for $v \geq c$, consist of a trading price equal to

the desired one, $\tilde{p}_1(v,c)$, and a nontrading price chosen to ensure that trade occurs (we have selected $p_0 = \tilde{p}_1(v,c) - v$, but any $p_0 = \tilde{p}_1(v,c) - k$ where $c \leq k \leq v$ would do). The final diagonal element has the nontrading price equal to the desired one, $\tilde{p}_0$ (since trade never occurs when $v < c$, the trading price is irrelevant -- here we have set it at $\tilde{p}_0$).

The off-diagonal elements are a bit more complicated. They are indicated in the diagram for the case where one party announces w_{t+1}. We now describe how they are determined in the case where one party announces w_i and the other w_j, where $i,j < t+1$, $i \neq j$.

Consider the sub-box (or sub-game) corresponding to the announcements w_i, w_j.

Seller

		w_i	w_j
Buyer	w_i	$(\tilde{p}_1(w_i)-v_i,\ \tilde{p}_1(w_i))$	$(p_0^{ij},\ p_1^{ij})$
	w_j	$(p_0^{ji},\ p_1^{ji})$	$(\tilde{p}_1(w_j)-v_j,\ \tilde{p}_1(w_i))$

Figure 5

Without loss of generality, suppose $\tilde{p}_1(w_j) \geq \tilde{p}_1(w_i)$. There are three cases:

Case (i) $\tilde{p}_1(w_j) = \tilde{p}_1(w_i)$.

Then choose $(p_0{}^{ij}, p_1{}^{ij}) = (p_0{}^{ji}, p_1{}^{ji}) = (\tilde{p}_0, \tilde{p}_1(w_j))$

Case (ii) $\tilde{p}_1(w_j) > \tilde{p}_1(w_i)$ and $v_j - v_i \geq c_j - c_i$.

Then choose $(p_0{}^{ij}, p_1{}^{ij}) = (\tilde{p}_1(w_j) - v_j, \tilde{p}_1(w_j))$

$(p_0{}^{ji}, p_1{}^{ji}) = (\tilde{p}_1(w_i) - v_i, \tilde{p}_1(w_i))$.

Case (iii) $\tilde{p}_1(w_j) > \tilde{p}_1(w_i)$ and $v_j - v_i < c_j - c_i$.

Then choose $(p_0{}^{ij}, p_1{}^{ij}) = (\tilde{p}_1(w_i) - c_i, \tilde{p}_1(w_i))$

$(p_0{}^{ji}, p_1{}^{ji}) = (\tilde{p}_1(w_j) - c_j, \tilde{p}_1(w_j))$.

We have now described how all the payoffs of the game are determined (if $\tilde{p}_1(w_j) < \tilde{p}_1(w_i)$, reverse i,j in cases (ii)-(iii) above). It remains to show that given any realized $(v_i, c_i) = w_i$, each party will tell the truth, i.e. send the message w_i -- so that the price $\tilde{p}_1(w_i)$, together with trade, is implemented if $v_i \geq c_i$, and the price $\tilde{p}_0$, together with no trade, is implemented if $v_i < c_i$.

Suppose first the realization is such that $v < c$. Then no trade occurs whatever messages are sent. If the buyer announces w_i, $i < t+1$, while the seller announces w_{t+1}, the buyer pays price $\tilde{p}_1(w_i) - c_i \geq \tilde{p}_0$ by condition (3) of the proposition. Hence a deviation by the buyer from the strategies (w_{t+1}, w_{t+1}) is not profitable. On the other hand, if the seller announces w_i, $i < t+1$, while the buyer announces w_{t+1}, the seller receives $\tilde{p}_i(w_i) - v_i \leq \tilde{p}_0$ by condition (3). Hence a deviation by the seller is also not profitable. It follows that (w_{t+1}, w_{t+1}) is a Nash equilibrium when $v < c$.

(Note that there may be other Nash equilibria; however since the game is zero-sum, they are all equivalent.)

Suppose next the realization is such that $v_i \geq c_i$. Consider first whether the buyer wants to deviate from "truth-telling", given that he expects the seller to announce w_i. If the buyer announces w_{t+1}, the price pair will be $(\tilde{p}_1(w_i) - v_i, \tilde{p}_1(w_i))$, and so, since $v_i \geq c_i$, trade will occur at price $\tilde{p}_1(w_i)$, which is also the ruling price if the buyer tells the truth. So a deviation to w_{t+1} is not profitable for the buyer. What about a deviation to w_j, where $j < t+1$? Then figure 5 applies, and the price pair is (p_0^{ji}, p_1^{ji}). To see that such a deviation is unprofitable, we separately consider the following cases:

<u>Case (a)</u>: $\tilde{p}_1(w_j) = \tilde{p}_1(w_i)$.

Trade occurs at price $\tilde{p}_1(w_j)$, and so the buyer gains nothing.

<u>Case (b)</u>: $\tilde{p}_1(w_j) > \tilde{p}_1(w_i)$ and $v_j - v_i \geq c_j - c_i$.

Trade occurs at price $\tilde{p}_1(w_i)$, and so the buyer gains nothing.

<u>Case (c)</u>: $\tilde{p}_1(w_j) > \tilde{p}_1(w_i)$ and $v_j - v_i < c_j - c_i$.

From condition (2) of the proposition, $c_j - c_i \geq \tilde{p}_1(w_j) - \tilde{p}_1(w_i) > 0$, so the seller wants to trade at prices $(\tilde{p}_1(w_j) - c_j, \tilde{p}_1(w_j))$. But the buyer may not (he won't if $v_i < c_j$). If the buyer does want to, the trading price will be $\tilde{p}_1(w_j)$, which exceeds $\tilde{p}_1(w_i)$, and so he will not have gained by his

deviation. If the buyer does not want to trade at prices $(\tilde{p}_1(w_j) - c_j, \tilde{p}_1(w_j))$, the contract will be renegotiated and the trading price will be $\tilde{p}_1(w_j) - c_j + v_i$. But from condition (3), this amount is at least $\tilde{p}_1(w_i)$. Hence the buyer's deviation is unprofitable.

<u>Case (d)</u>: $\tilde{p}_1(w_i) > \tilde{p}_1(w_j)$ and $v_i - v_j \geq c_i - c_j$.

Trade occurs at price $\tilde{p}_1(w_i)$, and so the buyer gains nothing.

<u>Case (e)</u>: $\tilde{p}_1(w_i) > \tilde{p}_1(w_j)$ and $v_i - v_j < c_i - c_j$.

From condition (2) of the proposition, $c_i - c_j \geq \tilde{p}_1(w_i) - \tilde{p}_1(w_j) > 0$, so the seller will not trade at prices $(\tilde{p}_1(w_j) - c_j, \tilde{p}_1(w_j))$. The contract will be renegotiated and the trading price will be $\tilde{p}_1(w_j) - c_j + c_i$, which (again by condition (2)) is at least $\tilde{p}_1(w_i)$. Hence the buyer's deviation is unprofitable.

We have established that in all cases, if the seller announces the truth, the buyer can do no better than announce the truth too. A similar argument shows that it does not pay the seller to deviate from the truth, if the buyer is not going to. Hence truth-telling is a Nash equilibrium if $v \geq c$ (again there may be other Nash equilibria, but they are all equivalent).

This proves proposition 3. Q.E.D.

B2. Proof of Proposition 5

Proposition 2 specifies what prices can be implemented when messages are verifiable. It will be helpful to simplify the notation a little.

First, for all $v_i \geq c_j$, define

$$P_{ij} \equiv P_1^*(v_i, c_j) - P_0^* .$$

Then, from parts (1) and (3) of Proposition 2, we know

$$P_{i,j+1} \leq P_{ij} \leq P_{i+1,j} \qquad \text{(monotonicity)}$$

$$\text{and} \quad c_j \leq P_{ij} \leq v_j . \qquad \text{(voluntary trade)}$$

Secondly, for all i and j, define

$$\Delta\pi_i \equiv \pi_i^+ - \pi_i^-$$

$$\text{and} \quad \Delta\rho_j \equiv \rho_j^+ - \rho_j^- .$$

Then $\Delta\pi_i/\pi_i(\beta)$ and $\Delta\rho_j/\rho_j(\sigma)$ are increasing in i and j. Notice that these imply first-order stochastic dominance.

Thirdly, in what follows, let Σ denote $\displaystyle\sum_{i=1}^{I}\sum_{j=1}^{J}$
$\quad v_i \geq c_j$

The buyer's net gain from marginally increasing β is

$$\Sigma\, \Delta\pi_i \rho_j(\sigma)\left[v_i - P_{ij}\right] - h_b'(\beta).$$

The first term is bounded. It is also nonnegative -- using stochastic dominance and the fact that $v_i - P_{ij}$ is nondecreasing in i (from Proposition 2(2)). So it follows from condition (3) of Proposition 5 that a necessary and sufficient condition for the buyer's optimal choice of β in $[0,1]$ is

$$\text{(B2.1)} \qquad \Sigma\, \Delta\pi_i \rho_j(\sigma)\left[v_i - P_{ij}\right] - h_b'(\beta) = 0.$$

Likewise, the seller's choice of σ can be summarised by

(B2.2) $$\Sigma\ \pi_i(\beta)\Delta\rho_j[P_{ij} - c_j] - h_s'(\sigma) = 0.$$

Consider the following relaxed programme (RP):

$$\text{Maximise } \Sigma\ \pi_i(\beta)\rho_j(\sigma)[v_i - c_j] - h_b(\beta) - h_s(\sigma)$$

subject to (B2.3) $\Sigma\ \Delta\pi_i\rho_j(\sigma)[v_i - P_{ij}] - h_b'(\beta) \geq 0$

(B2.4) $\Sigma\ \pi_i(\beta)\Delta\rho_j[P_{ij} - c_j] - h_s'(\sigma) \geq 0$

$\beta,\ \sigma \geq 0$

and monotonicity + voluntary trade.

(RP) is "relaxed" in two respects. First, the equalities (B2.1) and (B2.2) have been replaced by inequalities. This is just a technical device: in Lemma 3 below, their respective (nonnegative) Kuhn-Tucker multipliers γ and θ will be shown to be positive at an optimum, implying that the inequality constraints are binding. Secondly, the restriction on $P_1^*(v,c)$ in Proposition 2(2) has been omitted. The reason for this is that, as we will see, the trading prices which solve (RP) satisfy this restriction anyway.

Note that the level of P_0^* is left undetermined in (RP); this is because, with risk neutrality, it is equivalent to a transfer payment at date 0 which ensures that the contract is worth $\overline{U}$ to the seller.

The necessary first-order condition for β is

$$\Sigma\ \Delta\pi_i\rho_j(\sigma)[v_i - c_j] - h_b'(\beta) - \gamma h_b''(\beta) + \theta\ \Sigma\ \Delta\pi_i\Delta\rho_j[P_{ij} - c_j] \leq 0,$$

with equality if $\beta > 0$. Using (B2.3), we see that this implies

(B2.5) $$\Sigma\ \Delta\pi_i\rho_j(\sigma)[P_{ij} - c_j] - \gamma h_b''(\beta) + \theta\ \Sigma\ \Delta\pi_i\Delta\rho_j[P_{ij} - c_j] \leq 0.$$

Likewise the first-order condition for σ implies

(B2.6) $\quad \Sigma\, \pi_i(\beta)\Delta\rho_j[v_i - P_{ij}] - \theta h_s''(\sigma) + \gamma\, \Sigma\, \Delta\pi_i\Delta\rho_j[v_i - P_{ij}] \leq 0.$

The proof of the Proposition proceeds via the following three Lemmata.

Lemma 1. At a solution to (RP), if $\theta > 0$, then for $j < J$ and $v_i \geq c_j$,

$$P_{ij} = \max\{P_{i,j+1}, c_j\},$$

and if $\gamma < 0$, then for $i < I$ and $v_i \geq c_j$,

$$P_{ij} = \min\{P_{i+1,j}, v_i\}.$$

Proof. By symmetry, we need only prove the first half of the Lemma. Suppose it is not true: for some $v_i \geq c_j$,

$$P_{ij} = k^+ \text{ and } P_{i,j+1} = k^- \text{ where } k^+ \text{ exceeds } k^- \text{ and } c_j.$$

Let t be the minimum τ satisfying $P_{\tau j} = k^+$ and $v_\tau \geq c_j$. And let T be the maximum τ satisfying $P_{\tau,j+1} = k^-$. Then monotonicity and voluntary trade imply that $P_{\tau,j+1} < P_{\tau j} \leq v_\tau$ and $P_{\tau j} \geq k^+ > c_j$ for all τ in $\{t, \ldots, T\}$. And so without violating monotonicity or voluntary trade, we can, for each τ in $\{t, \ldots, T\}$, lower $P_{\tau j}$ by $\varepsilon_j > 0$ and raise $P_{\tau,j+1}$ by $\varepsilon_{j+1} > 0$ -- where the (small) ε_j, ε_{j+1} are chosen so as not to disturb the LHS of (B2.3):

$$\text{i.e. } \rho_j(\sigma)(-\varepsilon_j) + \rho_{j+1}(\sigma)\varepsilon_{j+1} = 0.$$

The effect on the LHS of (B2.4) is

$$\sum_{\tau=t}^{T} \pi_\tau(\beta)\,[\Delta\rho_j(-\varepsilon_j) + \Delta\rho_{j+1}\varepsilon_{j+1}] = \sum_{\tau=t}^{T} \pi_\tau(\beta)\rho_j(\sigma)\varepsilon_j\left[\frac{\Delta\rho_{j+1}}{\rho_{j+1}(\sigma)} - \frac{\Delta\rho_j}{\rho_j(\sigma)}\right]$$

-- which is positive. But the fact that we can slacken the constraint (B2.4) in this way contradicts $\theta > 0$.

Next a technical Lemma which will be of use later.

Lemma 2. If x_i is nondecreasing in i, and y_j is nonincreasing in j, then

$$\sum_{\substack{i=1 \\ x_i \geq y_j}}^{I} \sum_{j=1}^{J} \Delta\pi_i \Delta\rho_j [x_i - y_j] \geq 0,$$

and the inequality is strict if and only if $x_I - y_J > 0 > x_1 - y_1$.

Proof. Define $z_{ij} \equiv \max\{0, x_i - y_j\}$. It is straightforward to show that, for $i < I$ and $j < J$,

(B2.7) $$z_{i+1,j+1} - z_{i,j+1} - z_{i+1,j} + z_{ij} \geq 0,$$

with equality iff either $x_{i+1} \leq y_{j+1}$ or $x_i \geq y_j$.

For each i, define $\xi_i \equiv \sum_{i=1}^{J} \Delta\rho_j z_{ij}$.

Take a particular $i < I$. Now

$$\xi_{i+1} - \xi_i = \sum_{j=1}^{J} \Delta\rho_j [z_{i+1,j} - z_{ij}],$$

which from (B2.7) and stochastic dominance is non-negative, and zero iff either $x_{i+1} \leq y_{j+1}$ or $x_i \geq y_j$ for all $j < J$ -- i.e. iff either $x_{i+1} \leq y_J$ or $x_i \geq y_1$. Therefore, again from stochastic dominance, $\sum_{i=1}^{I} \Delta\pi_i \xi_i \geq 0$, with equality iff either $x_{i+1} \leq y_J$ or $x_i \geq y_1$ for all $i < I$ -- i.e. iff either $x_I \leq y_J$ or $x_1 \geq y_1$. Q.E.D.

Lemma 3. At a solution to (RP), $\gamma > 0$ and $\theta > 0$.

Proof. Suppose not: without loss of generality, suppose $\gamma = 0$.

First we show that $\theta > 0$. For if $\theta = 0$, then from (B2.5) and (B2.6),

$$\Sigma\ \Delta\pi_i \rho_j(\sigma)[P_{ij} - c_j] \leq 0,$$

$$\text{and}\quad \Sigma\ \pi_i(\sigma)\Delta\rho_j[v_i - P_{ij}] \leq 0.$$

But monotonicity, voluntary trade, and stochastic dominance together imply

that the LHSs of these inequalities are <u>nonnegative</u>, and equal zero if and only if

$v_I \geq c_j \Rightarrow P_{ij}$ is independent of i (and equal to c_j if $v_1 < c_j$)

and $c_J \leq v_i \Rightarrow P_{ij}$ is independent of j (and equal to v_i if $c_1 > v_i$).

Either $v_1 < c_J$ or $v_1 \geq c_J$. Therefore, either $P_{IJ} = c_J$ or $P_{IJ} = P_{1J} = v_1$. But a symmetric argument shows that either $P_{IJ} = v_I$ or $P_{IJ} = c_1$. But this contradicts I, J $\geq$ 2 and assumption (4) of the Proposition. Hence $\theta > 0$.

Consider the LHS of (B2.5). Monotonicity, voluntary trade, and stochastic dominance together imply that the first term is nonnegative. The second term is zero, since $\gamma = 0$. Hence the third term must be nonpositive. But, since $\theta > 0$, Lemma 1 tells us that, for those $v_i \geq c_j$, either $P_{ij} = c_j$ or P_{ij} equals some x_i, say, which is independent of j and (from monotonicity) nondecreasing in i. Hence Lemma 2 applies (setting $y_j \equiv c_j$), and the third term of the LHS of (B2.5) is non-negative, and zero only if $P_{ij} \equiv c_j$ for all $v_i \geq c_j$. But this last cannot be the case, since it would mean $P_{IJ} < \min \{v_I, P_{I,J-1}\}$ and therefore a first-order condition for P_{IJ}: $\theta\pi_I(\beta)\Delta\rho_J \leq 0$ -- which contradicts $\theta, \Delta\rho_J > 0$. Q.E.D.

Lemmas 1 and 3 together imply that in a second-best contract, the trading prices have the form given in Figure 2. These can be achieved with a simple two-price contract (P_0, P_0+k) without messages, as claimed in proposition 5. Note that the omitted restriction on trading prices given in proposition 2(2) is satisfied.

For clarity, denote the second-best levels of β,σ by $\hat{\beta},\hat{\sigma}$. It remains to show that $\hat{\beta} < \beta^*$ and $\hat{\sigma} < \sigma^*$.

Define $G(\beta,\sigma) \equiv \Sigma\, \pi_i(\beta)\rho_j(\sigma)[v_i - c_j]$

$$X(\beta,\sigma) \equiv \Sigma\, \pi_i(\beta)\rho_j(\sigma)[v_i - P_{ij}]$$

$$Y(\beta,\sigma) \equiv \Sigma\, \pi_i(\beta)\rho_j(\sigma)[P_{ij} - c_j].$$

Then $G_1(\beta^*, \sigma^*) = h_b'(\beta^*)$ and $G_2(\beta^*, \sigma^*) = h_s'(\sigma^*)$.

Also G_1, X_1, Y_1 are independent of β and G_2, X_2, Y_2 are independent of σ. Monotonicity, voluntary trade and stochastic dominance together imply $G_1 \geq X_1$, $X_2 \geq 0$, $G_2 \geq Y_2$, and $Y_1 \geq 0$. Finally, $G_{12} = \Sigma\Delta\pi_i\Delta\rho_j[v_i - c_j] > 0$ from Lemma 2.

Suppose $\hat{\sigma} \leq (<)\ \sigma^*$. Then

$h_b'(\beta^*) = G_1(\beta^*, \sigma^*) \geq (>)\ G_1(\beta^*, \hat{\sigma}) = G_1(\hat{\beta}, \hat{\sigma}) \geq X_1(\hat{\beta}, \hat{\sigma}) = h_b'(\hat{\beta})$.

Thus $\hat{\beta} \leq (<)\ \beta^*$. Since we have shown, in the first part of the proof of Lemma 3, that $(G_1 - X_1, G_2 - Y_2) \neq (0,0)$, it follows that $(\hat{\beta}, \hat{\sigma}) \neq (\beta^*, \sigma^*)$. By symmetry, then,

either $\hat{\beta} < \beta^*$ and $\hat{\sigma} < \sigma^*$

or $\hat{\beta} > \beta^*$ and $\hat{\sigma} > \sigma^*$.

We now use revealed preference to rule out the latter possibility[19]

Since (β^*, σ^*) is first-best,

$$G(\hat{\beta}, \hat{\sigma}) - G(\beta^*, \sigma^*) < h_b(\hat{\beta}) - h_b(\beta^*) + h_s(\hat{\sigma}) - h_s(\sigma^*)$$

which in turn is no more than $X(\hat{\beta}, \hat{\sigma}) - X(\beta^*, \hat{\sigma}) + Y(\hat{\beta}, \hat{\sigma}) - Y(\hat{\beta}, \sigma^*)$

since $\hat{\beta}$ is the buyer's choice if $\sigma = \hat{\sigma}$ and $\hat{\sigma}$ is the seller's choice if $\beta = \hat{\beta}$.

But $X(\hat{\beta}, \hat{\sigma}) + Y(\hat{\beta}, \hat{\sigma}) = G(\hat{\beta}, \hat{\sigma})$, and so

(B2.8) $G(\beta^*, \sigma^*) > X(\beta^*, \hat{\sigma}) + Y(\hat{\beta}, \sigma^*)$.

Now, using $X_2 \geq 0$, $Y_1 \geq 0$, if $\hat{\beta} > \beta^*$ and $\hat{\sigma} > \sigma^*$, the RHS of (B2.8) is not less than $X(\beta^*, \sigma^*) + Y(\beta^*, \sigma^*) = G(\beta^*, \sigma^*)$.

Contradiction. Proposition 5 is proved. Q.E.D.

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